Ecuaciones exponenciales y logarítmicas

1

a) $\frac{1}{e^x} = 27 \rightarrow \frac{1}{27} = e^x \rightarrow ln\ \frac{1}{27} = ln\ e^x$

$x = ln\ \frac{1}{27} = ln\ 1 - ln\ 27 = 0 - ln\ 27 \rightarrow x \simeq 3{,}296$

b) $e^{x-9} = \sqrt{73} \rightarrow ln\ e^{x-9} = ln\ \sqrt{73}$

$x - 9 = \frac{1}{2}\ ln\ 73 \rightarrow x = 9 + \frac{ln\ 73}{2} \rightarrow x \simeq 11{,}145$

c) $6^x = 81;\ x\ log\ 6 = log\ 81$

$x = \frac{log\ 81}{log\ 6} \approx 2{,}453$

d) $\frac{2^x}{3^x \cdot 3} = 1;\ \left(\frac{2}{3}\right)^x = 3;\ x\ log\ \frac{2}{3} = log\ 3$

$x = \frac{log\ 3}{log\ 2 - log\ 3} \approx -2{,}710$

2

a) $2^x + \frac{2}{2^x} = 3$

$z = 2^x \rightarrow z + \frac{2}{z} = 3;\ z^2 + 2 = 3z$

$z^2 - 3z + 2 = 0;\ z = \frac{3 \pm \sqrt{9-8}}{2} = \frac{3 \pm 1}{2} = \begin{matrix} 2 \\ 1 \end{matrix}$

$x_1 = 2;\ x_2 = 1$

b) $2 \cdot 2^x + \frac{2^x}{2} = \frac{5}{2};\ 4 \cdot 2^x + 2^x = 5;\ 2^x = 1$

$x = 0$

c) $2^{3+3x} + 2^{3x-1} = \frac{17}{16}$

$8 \cdot (2^x)^3 + \frac{(2x)^3}{2} = \frac{17}{16}$

$(128 + 8)\ (2^x)^3 = 17;\ (2^x)^3 = \frac{17}{136} = \frac{1}{8}$

$x = -1$

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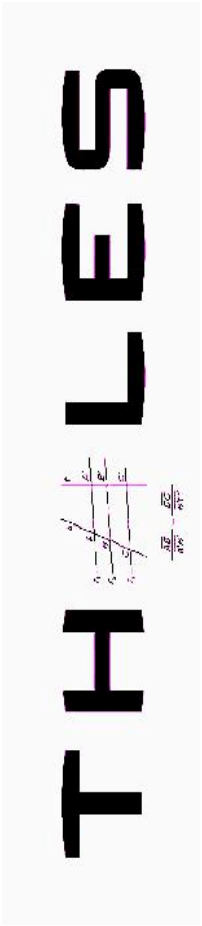

d) $(2^x)^2 - 5 \cdot 2^x + 4 = 0$

$2^x = \dfrac{5 \pm \sqrt{25 - 16}}{2} = \dfrac{5 \pm 3}{2} = \begin{cases} 4 \\ 1 \end{cases}$

$x_1 = 0;\ x_2 = 2$

e) $(3^x)^2 - 3^x - 6 = 0;\ 3^x = \dfrac{1 \pm \sqrt{1 + 24}}{2} = \dfrac{1 \pm 5}{2} = \begin{cases} 3 \\ -2 \text{ (no vale)} \end{cases}$

$x = 1$

f) $7 \cdot (7^x)^2 - 50 \cdot 7^x + 7 = 0;\ 7^x = \dfrac{50 \pm 48}{14} = \begin{cases} 7 \\ 1/7 \end{cases}$

$x_1 = -1;\ x_2 = 1$

3

a) $\log \dfrac{x^2 + 1}{x^2 - 1} = \log \dfrac{13}{12}$

$12x^2 + 12 = 13x^2 - 13;\ 25 = x^2$

$x_1 = -5;\ x_2 = 5$

b) $\ln (x^2 - 2x - 3) = \ln (3x - 3)$

$x^2 - 2x - 3 = 3x - 3;\ x^2 - 5x = 0$

$x = 5$ ($x = 0$ no vale)

c) $\ln (x - 3)^2 = \ln \dfrac{x}{4}$

$x^2 + 9 - 6x = \dfrac{x}{4}$

$4x^2 + 36 - 24x = x;\ 4x^2 - 25x + 36 = 0$

$x = \dfrac{25 \pm 7}{8} = \begin{matrix} 4 \\ 9/4 \text{ (no vale)} \end{matrix}$

$x = 4$

d) $\log \dfrac{x + 3}{x - 6} = 1$

$x + 3 = 10x - 60;\ 63 = 9x$

$x = 7$

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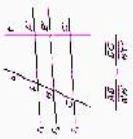
THALES



4 a) $\log \dfrac{x+9}{x} = 2$

$x + 9 = 100x;\ \ 9 = 99x;\ \ x = \dfrac{9}{99} = \dfrac{1}{11}$

$x = \dfrac{1}{11}$

b) $\dfrac{\log(x(3x+5))}{2} = 1;\ \ 3x^2 + 5x - 100 = 0$

$x = \dfrac{-5 \pm 35}{6} = \begin{matrix} 5 \\ -40/6 \text{ (no vale)} \end{matrix}$

$x = 5$

c) $\log x = \dfrac{-7 \pm \sqrt{49+72}}{4} = \dfrac{-7 \pm 11}{4} = \begin{matrix} 1;\ x_1 = 10 \\ -18/4 = -9/2;\ x_2 = 10^{-9/2} \end{matrix}$

d) $x^2 - 7x + 110 = 100;\ \ x^2 - 7x + 10 = 0$

$x = \dfrac{7 \pm \sqrt{49-40}}{2} = \dfrac{7 \pm 3}{2} = \begin{matrix} 5 \\ 2 \end{matrix}$

$x_1 = 2;\ \ x_2 = 5$

e) $\log \dfrac{x^2 + 3x + 36}{x+3} = 1$

$x^2 + 3x + 36 = 10x + 30;\ \ x^2 - 7x + 6 = 0$

$x = \dfrac{7 \pm \sqrt{49-24}}{2} = \dfrac{7 \pm 5}{2} = \begin{matrix} 6 \\ 1 \end{matrix}$

$x_1 = 1;\ \ x_2 = 6$

f) $ln\ x + ln\ 2x + ln\ 4x = 3$

$ln(x \cdot 2x \cdot 4x) = 3$

$ln(8x^3) = 3 \ \rightarrow\ 8x^3 = e^3 \ \rightarrow\ x^3 = \dfrac{e^3}{8}$

$x = \sqrt[3]{\dfrac{e^3}{8}} = \dfrac{e}{2} = \ \rightarrow\ x = \dfrac{e}{2}$